

Studies of QCD at LEP II

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Abstract. A combination of measurements of α_s in e^+e^- annihilation at LEP is presented. Distributions of various event-shape variables measured by ALEPH, DELPHI, L3 and OPAL at centre-of-mass energies from 41 GeV to 206 GeV are analysed in a common theoretical framework using $\mathcal{O}(\alpha_s^2)$ +NLLA predictions. The dominant theoretical uncertainties associated to missing higher orders are studied in detail.

1 Introduction

The most important parameter of QCD, the strong coupling constant α_s , has been determined at LEP in different processes and with various observables over a wide range of energies. Here measurements using event-shape variables provided by the four LEP experiments are combined. Six observables are analysed, these are thrust, heavy jet mass, wide and total jet broadening, C-parameter and the three-jet resolution parameter $-\ln y_3$. The data have been collected at the LEP II-energies in the range from 133 GeV to 206 GeV, at the Z boson peak and at lower energies from 41 GeV to 85 GeV using radiative events. The combination of the measurements from different variables, energies and experiments takes into account correlations between the measurements. Theoretical uncertainties are re-evaluated for the individual input measurements and propagated to the combined values. This combination is performed by the LEP QCD working group [1].

2 Input measurements

The latest, partially still preliminary measurements of α_s provided by ALEPH [2], DELPHI [3], L3 [4] and OPAL [5] serve as input data for the combination. Altogether 194 input values enter the combination procedure. The measurements are consistently extracted from the event-shape distribution using a theoretical description recommended by the LEP QCD working group [1]. Statistical and experimental systematic uncertainties are evaluated by the experiments. In order to study hadronisation uncertainties the experiments have provided measurements using three generators PYTHIA, HERWIG and ARIADNE for hadronisation corrections. The dominant uncertainty is perturbative, and it is highly correlated between the input measurements. A special treatment is applied to calculate the theoretical uncertainty for each input measurement, as explained in the next section.

3 Theoretical predictions and uncertainties

3.1 Predictions

To second order in α_s , the distribution of a generic event-shape variable y ($y = 1 - T, \rho, B_W, B_T, y_3, C$) is given by:

$$\frac{1}{\sigma_{tot}} \frac{d\sigma(y)}{dy} = \bar{\alpha}_s(\mu^2) A(y) + (\bar{\alpha}_s(\mu^2))^2 \times \left[A(y) 2\pi b_0 \ln \left(\frac{\mu^2}{s} \right) + B(y) \right],$$

where $\bar{\alpha}_s = \frac{\alpha_s}{2\pi}$ and $b_0 = \frac{33-2n_f}{12\pi}$. The resummed calculations are applied to the cumulative cross section

$$R(y, \alpha_s) \equiv \frac{1}{\sigma_{tot}} \int_0^y \frac{d\sigma(x, \alpha_s)}{dx} dx.$$

The fit function consists of two components, the fixed order term and an expression resumming leading and next-to-leading logarithms to all orders in α_s . The recommended [6] prescription to merge the two calculations and to subtract off double counting terms is the Log(R) matching scheme

$$\ln R(y, \alpha_s) = L g_1(\alpha_s L) + g_2(\alpha_s L) - (G_{11} L + G_{12} L^2) \bar{\alpha}_s - (G_{22} L^2 + G_{23} L^3) \bar{\alpha}_s^2 + \mathcal{A}(y) \bar{\alpha}_s + \left[\mathcal{B}(y) - \frac{1}{2} \mathcal{A}^2(y) \right] \bar{\alpha}_s^2,$$

with $L = \ln(y_0/y)$ where $y_0 = 1$ for $y = 1 - T, \rho, y_3, B_T, B_W$ and $y_0 = 6$ for C . An improved version, the modified Log(R) scheme is used in the fit functions to determine α_s , which vanishes at a given phase space limit y_{max} . To fulfil this constraint L is replaced by

$$\tilde{L} = \frac{1}{p} \ln \left(\left(\frac{y_0}{y} \right)^p - \left(\frac{y_0}{y_{max}} \right)^p + 1 \right),$$

with the modification degree power $p = 1$. A detailed collection of the formulae and numerical values for y_{max} and the matching coefficients are given in [6].

3.2 Uncertainties

Theoretical uncertainties stemming from unknown higher orders in the perturbation series are inherently difficult to assess. The method adopted here includes a variation of the renormalisation scale x_μ from 0.5 to 2 and a new test which re-scales the logarithmic variable $L \rightarrow \ln \frac{y_0}{x_L y}$ in the range $2/3 < x_L < 3/2$, as suggested in [6]. In addition a different matching scheme and different implementations of the kinematic modification are tested. Different sources of the perturbative uncertainty are combined with the uncertainty band method. This method calculates the uncertainty for a given variable and experimentally used fit range with a common value of $\alpha_s(M_Z)$, the latter being the result of a first combination iteration of all input measurements. The uncertainties of the distributions are calculated first, taking the prediction of the modified Log(R) matching scheme as reference theory and constructing an uncertainty band from the largest deviation with respect to the reference in each bin out of the theoretical variants mentioned above. In a second step the reference theory is calculated with variable α_s and the smallest resp. largest value yielding a prediction inside the uncertainty band defines the theoretical uncertainty of α_s . The method is illustrated in fig. 1 taking C-parameter as example. The

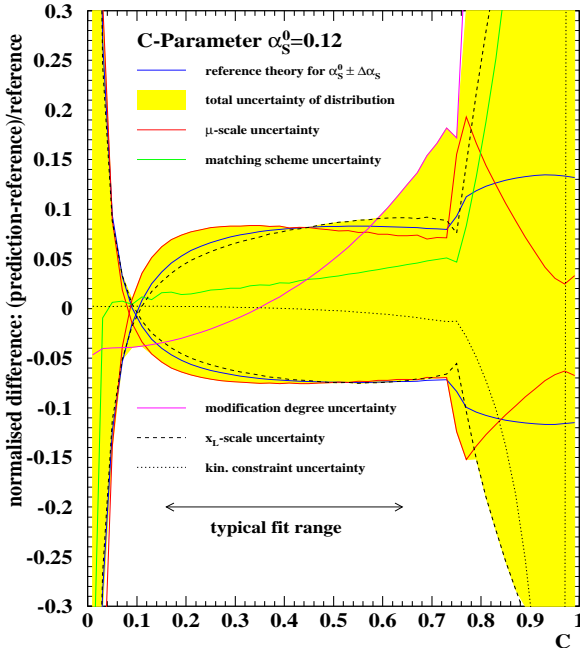


Fig. 1. Theoretical uncertainties for C-parameter. The grey area corresponds to the total perturbative uncertainty, dotted and dashed lines show individual contributions. The full lines represent reference predictions with values of α_s shifted by its uncertainty.

main two components to the uncertainty are the variations of the scales x_μ and x_L , which contribute to a similar amount but in different regions of the distributions.

4 Combination procedure

The procedure to combine the 194 measurements begins with the construction of a 194×194 covariance matrix V , which relates the uncertainties for all pairs of input measurements. The covariance matrix is decomposed into four components corresponding to the four sources of errors:

$$V_{ij}^{\text{total}} = V_{ij}^{\text{stat.}} + V_{ij}^{\text{exp.}} + V_{ij}^{\text{had.}} + V_{ij}^{\text{th.}}$$

The best combined value $\hat{\alpha}_s$ yielding the minimal error is a weighted average with weights obtained as follows

$$\hat{\alpha} = \sum_i w_i \alpha_s^i, \quad \text{where} \quad w_i = \frac{\sum_j (V^{-1})_{ij}}{\sum_{jk} (V^{-1})_{jk}}.$$

The uncertainty of the combination is given for each component as function of the weights, e.g. for the statistical uncertainty $\sigma_{\text{stat.}}^2 = \sum_{ij} w_i V_{ij}^{\text{stat.}} w_j$. A specific treatment of the correlation of each uncertainty component concerning the off-diagonal matrix elements of V is applied.

The statistical uncertainties are uncorrelated between different experiments and between different energies, but correlated for different observables using the same events. These correlations have been estimated numerically, using a large number of simulated datasets.

The experimental systematic uncertainties are considered to be uncorrelated between different experiments. Between different energies and/or different observables of the same experiment, the “minimum overlap” assumption is made: $V_{ij}^{\text{exp.}} = [\min(\sigma_i^{\text{exp.}}, \sigma_j^{\text{exp.}})]^2$.

Studies of the LEP QCD working group revealed that the hadronisation uncertainty is largely uncorrelated between experiments because of different tunings and versions of the generators. Therefore, the off-diagonal matrix elements of $V^{\text{had.}}$ are set to zero and the hadronisation uncertainty is determined by repeating the combination procedure for each of the generators separately. The nominal results uses PYTHIA for corrections and as uncertainty the standard deviation of the three combinations is taken. For the combinations by energy the raw hadronisation uncertainty is fitted by a power law form $a + b/Q$ in order to suppress statistical fluctuations.

The theory uncertainties are expected to be highly correlated between all pairs of measurements of the same observable and to varying extents between measurements of different observables. The exact determination of the correlation, however, turned out to be difficult. Various attempts indicated correlation coefficients of the order of 90%, but the result of the combination, in particular its theoretical uncertainty, depends strongly on the assumptions made. A more reliable approach is chosen, setting again the off-diagonal matrix elements of $V^{\text{th.}}$ to zero and repeating the combination for two additional alternative input values of $\alpha_s^0 \pm \Delta\alpha_s^{\text{th.}}$, where $\alpha_s^{\text{th.}}$ is obtained with the uncertainty band method. As perturbative uncertainty the difference between the nominal combined result and the alternative combinations is taken.

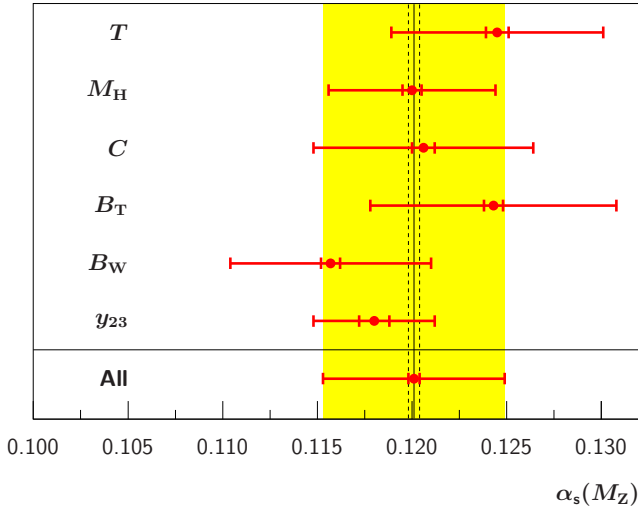
Table 1. Combinations of $\alpha_s(M_Z)$ for different energy ranges.

	$Q < M_Z$	$Q = M_Z$	$Q > M_Z$	All
$\alpha_s(M_Z)$	0.1208	0.1200	0.1201	0.1201
stat. error	± 0.0012	± 0.0002	± 0.0005	± 0.0003
exp. error	± 0.0023	± 0.0008	± 0.0010	± 0.0009
had. error	± 0.0032	± 0.0010	± 0.0007	± 0.0009
th. error	$+0.0052$ -0.0050	$+0.0048$ -0.0048	$+0.0044$ -0.0045	$+0.0046$ -0.0047
total error	± 0.0066	± 0.0050	± 0.0047	± 0.0048

5 Results

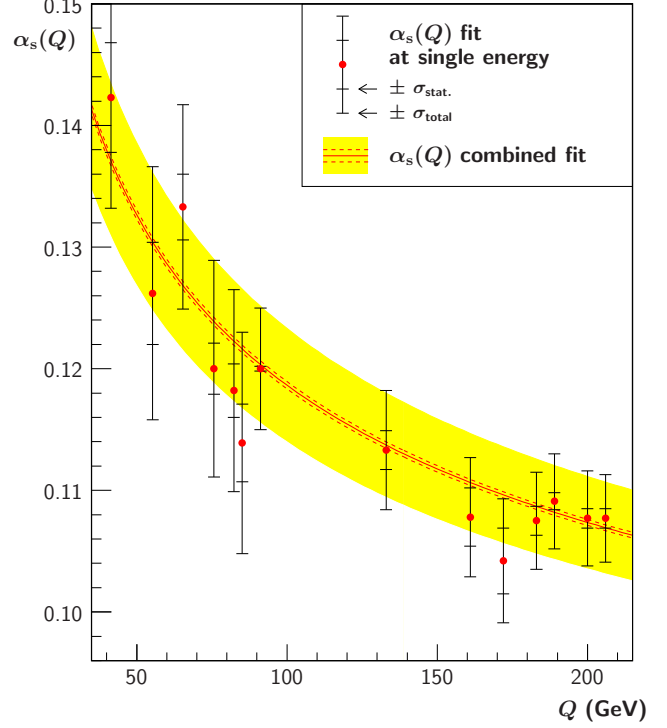
The combination is carried out for different subsets of variables and energies, the full results are given in [1]. The main result is summarised in Table 1, where all variables and experiments are combined for different ranges in energy. The best combination with the smallest total uncertainty is obtained with the LEP II data alone, combining all data yields a larger uncertainty. This is mainly due to the theoretical uncertainty, which is decreasing with increasing energy, scaling with α_s^3 . The hadronisation uncertainty is decreasing as well, scaling with $1/Q$.

The perturbative uncertainties are the key element limiting the precision of present elements, and their exact determination is cumbersome. As a cross check, results from different variables are compared in fig. 2. As each variable receives possibly different higher order contribution, the spread of results in α_s indicates their size. The RMS between results from different variables is 0.0016 with a maximum spread of 0.0046, in good agreement with the perturbative uncertainty of ± 0.0047 for the combination of all data.

**Fig. 2.** Combinations of α_s per variable.

In order to study the energy evolution of $\alpha_s(Q)$ the input measurements from all variables and experiments

are combined per energy Q . The result is shown in fig. 3, and compared to a the 3-loop evolution fit, which is found in excellent agreement with the data ($\chi^2/N_{dof}=11.7/13$).

**Fig. 3.** Energy evolution of $\alpha_s(Q)$.

6 Conclusions

A preliminary combination of measurements of α_s from the LEP experiments using event-shape variables is presented. The values of α_s have consistently been determined in a common theoretical framework and a new method is applied to assess perturbative uncertainties. The combined results are consistent with the expected energy evolution of α_s . The preliminary result using all LEP data is $\alpha_s(M_Z) = 0.1201 \pm 0.0048$.

References

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